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**INTERACTIVE AIRCRAFT FLIGHT CONTROL AND
AEROELASTIC STABILIZATION**

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Introduction

The purpose of this study is to examine the potential benefits and costs of optimizing both the structural stiffness and the active control of aircraft in a rational manner. The ultimate goal of this effort is to arrive at a unified treatment of structural and active control design for the stability augmentation of flexible aircraft.

Three separate efforts have taken place during the past six months of effort. The first effort is an exhaustive literature evaluation in the area of passive tailoring for aircraft performance. During this effort, several valuable and previously unrecognized tailoring studies were uncovered. This survey was combined with similar work by Messrs. M.H. Shirk and T.J. Hertz of the Air Force Wright Aeronautical Laboratories to produce a paper presented at the 25th AIAA Structures, Structural Dynamics and Materials Conference in Palm Springs, California in May 1984.

The second effort involved the identification of a mathematical technique to be used for aeroservoelastic tailoring studies. A promising candidate method has been identified and is described in the following section.

Finally, two analytical models, one elementary, the other sophisticated, have been developed to illustrate the potential for aeroservoelastic tailoring. Both models have essential features of "real-world" hardware, yet the physical understanding is not buried in a myriad of detail. These models are also described in the next section.

The Use of Structural Gains as Design Parameters

There are two major obstacles to simultaneous treatment of the structural stiffness design optimization problem and the active controls problem. The first difficulty arises because of the dissimilarity of design variables in the two problems. This difficulty has been overcome, at least at the elementary level, by the selection of a characteristic set of nondimensional parameters for beam-like and plate-like structures. The state space model of an aeroelastic system can be written as:

$$\dot{x} = Ax + Bu \quad (1)$$

$$y = Cx \quad (2)$$

with x as the n -dimensional state vector, u is an m -dimensional control vector and y is the output vector, while A, B and C are constant coefficient matrices. If a linear, full-state feedback control law exists, of the form,

$$u = -Gx \quad (3)$$

then the modified system equations are:

$$\dot{x} = (A - BG)x \quad (4)$$

On the other hand, the equations for a structural system with passive control may be written as:

$$\dot{x} = Ax - \psi \bar{A}x \quad (5)$$

where ψ is a nondimensional parameter related to stiffness cross-coupling provided by structural tailoring and \bar{A} is a modification to the A matrix provided by changes in the stiffness matrix.

Equation 5 resembles Eqn. 6 in that

$$\psi \bar{A}x = BGx \quad (6)$$

If there are several variables, ψ_i , corresponding to tailored bays of a wing for instance, Eqn. 6 becomes

$$\sum \psi_i \bar{A}_i x = BGx \quad (7)$$

Theoretically, one should be able to construct a structural modification in terms of $\psi_i \bar{A}_i$ to furnish the same equivalent (in terms of eigenvalues) system as the actively controlled system. A major problem arises, however, because the elements of \bar{A} are not free parameters while the elements of G are. Thus, standard optimal control procedures (for instance, pole placement) do not have an obvious adaptation. Attempts at such adaptations over the past six months have not proved productive.

Fortunately, a method developed by Newson and Gilbert offers at least a preliminary approach to the simultaneous design problem. If the cross-coupling parameter ψ is treated as a design parameter that is held fixed during the control design, its effect on the control system performance can be assessed by employing optimal sensitivity techniques. With this technique, a cost functional, J , is minimized to obtain the "optimal" control law for the system. The parameter ψ is then treated as a design variable so that the change in J with respect to ψ can be computed using an adaptation of the Newson/Gilbert approach. This adaptation is described in the Appendix to this report. In addition, the sensitivity of other aspects of the control law design to ψ may be assessed.

The "important", or at least new, aspect of this development is that a technique is available to assess the impact of structural tailoring upon active control design. The models on which this study will focus are described in the next section.

An Elementary Model for Aeroservoelastic Optimization

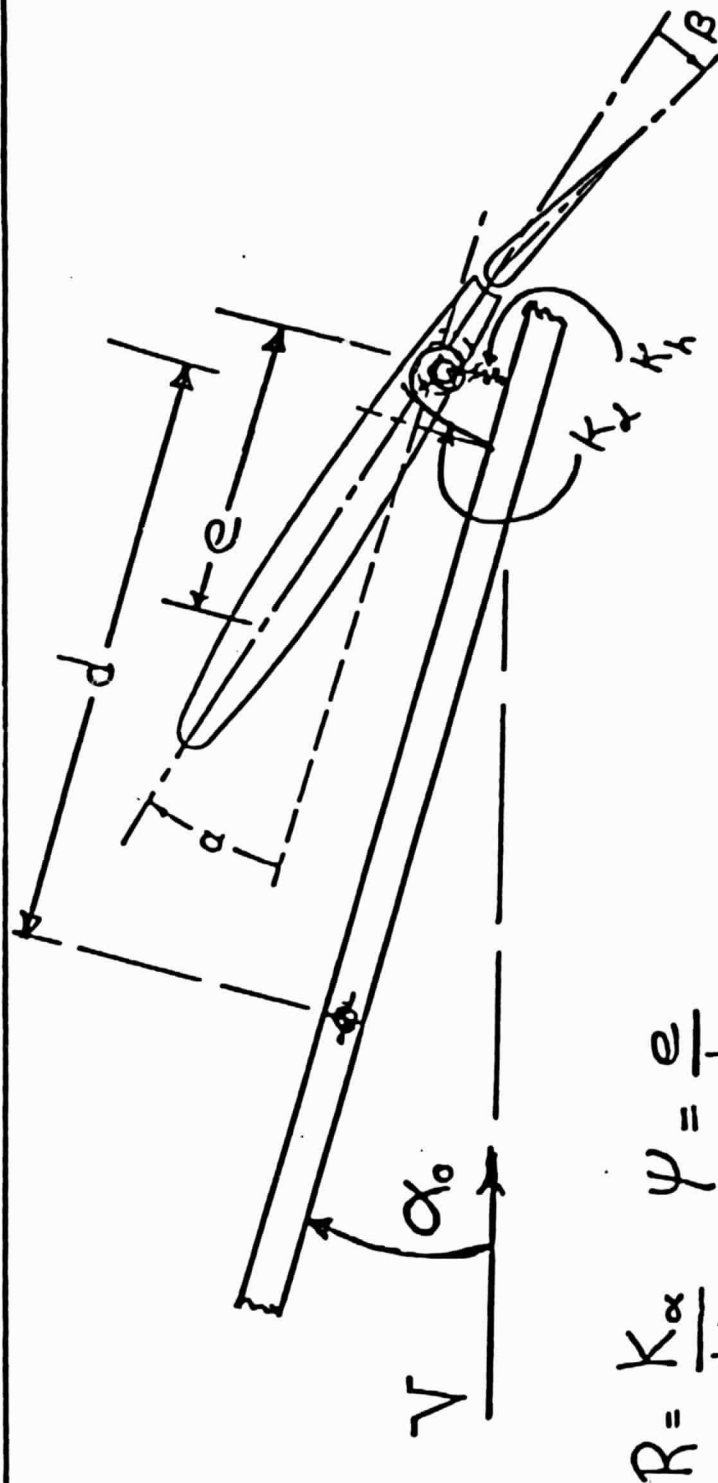
To study the problem of aeroservoelastic optimization, one may begin either at an advanced or an elementary level. The operative term would be "state-of-the-art." Reviews of the literature and past experience have convinced us that a first look at aeroservoelastic optimization (ASEO) should begin with an example that is simplistic, but meaningful. The model chosen is shown in Figure 1. This model consists of a typical section free to pitch and plunge as a rigid body. The design variable ψ is, in this case, equal to e/b . The dimension e/b measures the distance between the static aerodynamic center at the quarter-chord and the plunge spring position on the airfoil. For a fixed ratio $R = K_\alpha/K_h$ and with the airfoil c.g. position fixed, the divergence speed of the fixed root airfoil declines with increasing e/b . On the other hand, the flutter speed increases with increasing e/b . This provides a design trade-off situation for which an optimum value of e/b exists to maximize the aeroelastic stability of the system.

If the airfoil is attached to a fuselage element that is, in turn, free to pitch and plunge, the situation becomes more interesting because the value of e now determines the attitude stability of the aircraft and values of e that maximize the stability of the fuselage/wing combination may differ significantly from those which were found for the wing alone.

The addition of the control surface to the model provides additional design options. With R fixed the control effectiveness is unchanged by changes in e/b . Thus any design benefit or degradation is unrelated to control effectiveness in this idealization.

For fixed values of the system structural and inertial parameters, an optimal control law may be generated. A sensitivity analysis will then

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$$R = \frac{K_\alpha}{K_h}$$

$$\psi = \frac{e}{b}$$

Figure 1 2-D ASE0 Model

be performed to assess the effect of a change in e/b ($\delta e/b$) on the active control of the vehicle. From this information, a new value of e will be selected, together with new control parameters. One possible limiting case of this procedure is that the active control could disappear entirely, meaning that passive control is sufficient to handle the stability problem.

This model has most of the structural dynamic characteristics of an actual vehicle. The potential for strong rigid body/wing interaction exists, as does the capability of studying the differences between controlling the stability of the wing itself (in a fixed fuselage condition) and the wing/fuselage combination. The most serious limitation of this model is the limited number of degrees of freedom.

This analytical model does have advantages. It is a valuable learning tool, uncluttered by a myriad of numbers. Each step of the ASE0 procedure is easily understood and interpreted in light of the substantial amount of information available on 2-D sections.

Because of the limitations of the 2-D model, the ASE0 procedure will be extended to a realistic wing with a structurally tailored span. In this case the cross-coupling parameter ψ is the design variable. As before, the objective will be to improve overall performance in light of lessons learned with the 2-D model.

Appendix

Optimal Control and Sensitivity Derivatives for the Redesign Problem

The aeroelastic equations of motion may be written

$$\dot{X} = AX + Bu \quad . \quad (B.1)$$

If the control is a linear, measurement feedback control, then

$$u = GMX \quad (B.2)$$

where, M = state measurement matrix (i.e. $z = MX$)

and G = feedback gain matrix.

Then the control-augmented system matrix is

$$A_+ = A + BGM, \text{ so} \quad (B.3a)$$

$$\dot{X} = A_+ X \quad . \quad (B.3b)$$

The subscript "plus" sign denotes augmentation.

A quadratic cost function, used in linear regulator design, is

(21)

$$J = \int_0^{\infty} [X^* C^* Q C X + u^* R u] dt \quad (B.4)$$

where, C = output matrix (i.e. $y = CX$),

Q = output weighting matrix,

and R = control weighting matrix .

The well known solution for the optimal control that minimizes J , subject to the constraint, eqn. B.1, is

$$u = -R^{-1}B^*PX \quad (B.5a)$$

$$\text{or,} \quad GM = -R^{-1}B^*P \quad (B.5b)$$

where P is the solution to the steady-state matrix Riccati equation,

$$PA + A^*P + C^*QC - PBR^{-1}B^*P = 0 \quad (B.6)$$

For computation of sensitivity derivatives, it is assumed that the optimal control, eqn. B.5a, has been determined for a baseline configuration and that the weighting matrices used in this determination, Q and R, are "frozen" (i.e. insensitive to the design parameters, p_i , so that $\frac{\partial Q}{\partial p_i} \equiv 0$, and $\frac{\partial R}{\partial p_i} \equiv 0$). Also, the control input matrix, B, is considered to be dependent upon the type and geometry of the control being used, and not upon the design parameters. So, $\frac{\partial B}{\partial p_i} \equiv 0$, also. Since the cost function, defined in eqn. B.4, is what determines the optimality of the control design, it will also be the measure by which subsequent redesigns are judged.

First, the cost function is decomposed into its regulation and control parts,

$$J = J_x + J_u \quad (B.7)$$

Now,

$$J_x = X_0^* S_x X_0 \quad (B.8a)$$

where S_x satisfies

$$S_x A_+ + A_+^* S_x + C^*QC = 0 \quad (B.8b)$$

and

$$J_u = X_0^* S_u X_0 \quad (B.9a)$$

where S_u satisfies

$$S_u A_+ + A_+^* S_u + PBR^{-1}B^*P = 0 \quad . \quad (B.9b)$$

X_0 is the initial condition (time, t , is zero) on the state vector.

The regulation cost sensitivity with respect to p_i is found by differentiating eqns. B.8a and B.8b so that

$$\frac{\partial J_x}{\partial p_i} = X_0^* \frac{\partial S_x}{\partial p_i} X_0 \quad (B.10a)$$

where $\frac{\partial S_x}{\partial p_i}$ satisfies

$$\frac{\partial S_x}{\partial p_i} A_+ + A_+^* \frac{\partial S_x}{\partial p_i} + (S_x \frac{\partial A_+}{\partial p_i} + \frac{\partial A_+^*}{\partial p_i} QC + C^*Q \frac{\partial C}{\partial p_i}) = 0 \quad . \quad (B.10b)$$

Similarly, the control cost sensitivity can be found from

$$\frac{\partial J_u}{\partial p_i} = X_0^* \frac{\partial S_u}{\partial p_i} X_0 \quad (B.11a)$$

where $\frac{\partial S_u}{\partial p_i}$ satisfies

$$\frac{\partial S_u}{\partial p_i} A_+ + A_+^* \frac{\partial S_u}{\partial p_i} + (S_u \frac{\partial A_+}{\partial p_i} + \frac{\partial A_+^*}{\partial p_i} S_u + \frac{\partial P}{\partial p_i} BR^{-1}B^*P + PBR^{-1}B^* \frac{\partial P}{\partial p_i}) = 0 \quad (B.11b)$$

Then, in general, any desired change in the costs can be effected within the theoretical limits of the parameters, p_i (and provided there are a sufficient number, NP , of parameters), as

$$\Delta J_x = \sum_{i=1}^{NP} X_0^* \frac{\partial S_x}{\partial p_i} X_0 \Delta p_i \quad . \quad (B.12a)$$

and
$$\Delta J_u = \sum_{i=1}^{NP} X_0^* \frac{\partial S}{\partial p_i} X_0 \Delta p_i \quad . \quad (B.12b)$$

Since only first order derivatives are being used, it would be wise if the Δp_i 's are kept small throughout the redesign iterations.

To complete this derivation, it is necessary to obtain expressions for $\frac{\partial A_+}{\partial p_i}$ and $\frac{\partial P}{\partial p_i}$, found in eqns. B.10b and B.11b, which are as yet undetermined. By first differentiating the Riccati equation, eqn. B.6, and using the definition for A_+ , eqn. B.3a, and the solution for GM, eqn. B.5b, an equation that can be solved for $\frac{\partial P}{\partial p_i}$ can be obtained, namely,

$$\frac{\partial P}{\partial p_i} A_+ + A_+^* \frac{\partial P}{\partial p_i} + \left(\frac{\partial C^*}{\partial p_i} Q C + C^* Q \frac{\partial C}{\partial p_i} + P \frac{\partial A}{\partial p_i} + \frac{\partial A^*}{\partial p_i} P \right) = 0 \quad . \quad (B.13)$$

Equation B.13 is similar to one derived in (22) except that the Q, R, and B matrices are assumed insensitive to p_i , and the output matrix, C, is included explicitly. Note that $\frac{\partial A}{\partial p_i}$ is known (see sections 2 and 3). It is assumed that $\frac{\partial C}{\partial p_i}$ is known.* Then, $\frac{\partial A_+}{\partial p_i}$ can be found from

$$\frac{\partial A_+}{\partial p_i} = \frac{\partial A}{\partial p_i} - B R^{-1} B^* \frac{\partial P}{\partial p_i} \quad . \quad (B.14)$$

* The output matrix, C, can either be insensitive (i.e. $\frac{\partial C}{\partial p_i} \equiv 0$), or be some other known function of the parameter. For instance, if the output to be regulated, $y = CX$, consists of internal structural loads, then C will resemble some portion of the structural stiffness matrix.